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DEVELOPABLE WINGS

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DEVELOPABLE WINGS*

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In this work, we shall develop a method of computing aerodynamic characteristics of wings of finite span, of a type we shall call developable.

The whole study shall proceed under the assumption that the gas is devoid of forces of internal friction and moves adiabatically. We shall assume that the strong disruption surfaces do not recoil from either the frontal or the rear edges of the wing. The motion shall be assumed to be fixed. The interference of the wing with the fuselage shall be left out of the examination.

1. Developable Wings

We shall consider wing finite span of EFGH, streamlined by the supersonic flow of the gas, to be developable if:

1) the upper and lower surfaces of the wing are developable surfaces;
2) the frontal edge of the wing (EH) is a straight line;
3) the rear edge of the wing (FG) and the consolic edge (EF) are such that any Mach line traced forward from their points intersect the airfoil only at points on the frontal edge.

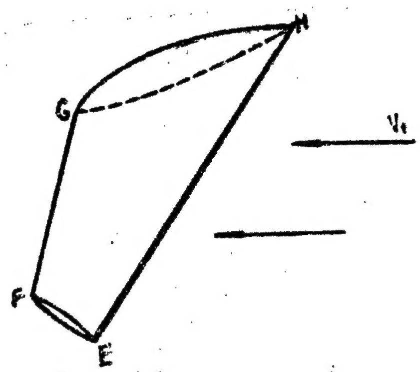

Fig. 1.

2. Generalization of the Prandl-Mayer Flow

[x]Please see page -a-

The following fact is well known from geometry: for any developable surfaces (except cones and cylinders), a curve of double curvature can be found having the property that the envelope of the tangent planes to the curve will yield the initial developable surface S.

This purely geometrical fact allows for an easy generalization of the flows of Prandtl-Mayer, and enables us to obtain analogous flows about any arbitrarily chosen, convex, developable surface -- including cones and cylinders.

Indeed, let us examine an arbitrary developable surface (Fig. 2). We find on the basis of the geometrical fact cited that it can be approximated to within any degree of exactness by a surface composed of flat strips, provided the number of such strips is sufficiently large. Cones and cylinders shall be temporarily excluded from consideration. The possibility of such an approximation for cones and cylinders does not follow from the geometrical fact stated but is directly evident.

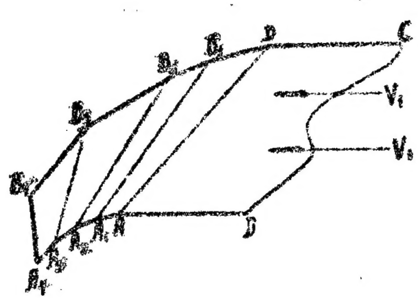

Fig. 2.

Thus, we may maintain that any developable surface can be approximated by a surface composed of flat strips, provided their number is sufficiently large.

We shall consider a convex developable surface which is flat to the right from the rectilinear generatrix (AB) and curvilinear to the left of same. We shall assume that a gas flows along the flat part of this surface with supersonic velocity $\bar{V}_1$ with constant hydrodynamic elements. The supersonic velocity $\bar{V}_1$ should be such that the projection of $\bar{V}_1$ on the normal to the generatrix AB traced in the plane ABCD remains supersonic.

We shall try to solve the problem of the movement of the gas about the curvilinear part of the developable surface under consideration.

For this purpose, we shall approximate the developable surface by a surface consisting of flat strips. The process of solving the problem becomes quite clear.

Indeed, the streamlining mechanism of a surface composed of flat strips (Fig. 2) is very simple. The hydrodynamic elements on such strips are constant, while at the edges AB, $A_1 B_1$, A_2B_2..., we always have the well-known sliding streamlining over an evolving angle. Here the projection of the velocity upon the edge of the angle remains constant, and has no meaning for streamlining.

Let us trace a plane Π along such an edge and the velocity vector.

We denote by V_τ the projection of the velocity on the direction of the edge, and by V_n the projection of the velocity on the normal to the edge that lies in the plane Π.

We shall further provide with the index 1 the values of all hydrodynamic elements prior to streamlining across the angle, and those streamlining past with the index 2. The angle of rotation of the gas flow shall be denoted by $\Delta\beta$. Then, to within errors amounting only to the higher powers of small magnitudes, we shall have

$$
\begin{gathered}
V_{\tau,2} = V_{\tau,1}, \\
V_{n,2} = V_{n,1} + \frac{a_1 V_{n,1}}{\sqrt{V_{n,1}^2 - a_1^2}} \Delta\beta, \qquad (2,1) \\
\frac{1}{2}\left(V_n^2 + V_\tau^2\right) + \frac{a^2}{k-1} = i_0,
\end{gathered}
$$

where a is the velocity of sound, i_0 the constant in Berboulli's formula and k the exponent of the adiabatics.

A subsequent application of the equations (2,1) and a corresponding sequence of projections on the changing directions of the axes n and τ permit to compute the gas flow to within any degree of precision desired.

However, it is more convenient to obtain the ordinary differential equations for V_τ and V_n.

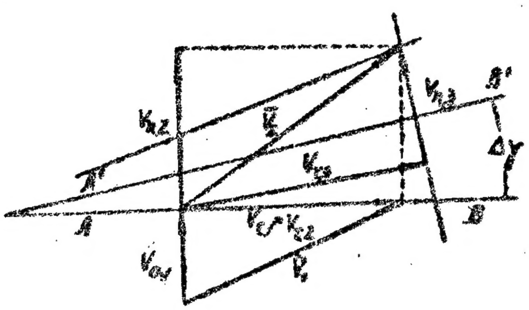

Fig. 3.

To derive these equations we shall examine Fig. 3. Let the edge AB be subject to a flow possessing velocities $V_{n,1}$ and $V_{\tau,1}$. Let us suppose that the angle of rotation of the flow about the edge AB is $\Delta\beta$ and let the angle between the edges AB and A'B' be $\Delta\gamma$. Then for the velocities $V_{n,3}$ and $V_{\tau,3}$ with which the gas flow approaches the edge A'B' we shall have the equations:

$$
\begin{gathered}
V_{n,3} = V_{n,2}\cos\Delta\gamma - V_{\tau,2}\sin\Delta\gamma, \qquad (2,2) \\
V_{\tau,3} = V_{\tau,2}\cos\Delta\gamma + V_{n,2}\sin\Delta\gamma.
\end{gathered}
$$

On substituting $V_{n,2}$ and $V_{\tau,2}$ from (2,1) into (2,2) and replacing sines by angles and cosines by units, we obtain:

$$
\begin{gathered}
V_{n,3} - V_{n,1} = \frac{a_1 V_{n,1}}{\sqrt{V_{n,1}^2 - a_1^2}} \Delta\beta - V_{\tau,1}\Delta\gamma, \\
V_{\tau,3} - V_{\tau,1} = V_{n,1}\Delta\gamma. \qquad (2,3)
\end{gathered}
$$

On dividing poth parts of the equalities (2,3) by $\Delta\beta$ and letting $\Delta\beta$ approach zero, we obtain the differential equations required.

$$\frac{dV_n}{d\beta} = \frac{aV_n}{\sqrt{V_n^2 - a^2}} - V_\tau \frac{d\gamma}{d\beta},$$

$$\frac{dV_\tau}{d\beta} = V_n \frac{d\gamma}{d\beta}. \qquad (2,4)$$

To these equations we must still adjoin Bernoulli's integral:

$$\frac{1}{2}(V_n^2 + V_\tau^2) + \frac{a^2}{k-1} = i_0. \qquad (2,5)$$

If the equation of the developable surface is given, then the function $d\gamma/d\beta$ is a completely defined function of the variable β. Moreover, the parameters of the gas flowing along the flat part of the surface provide us with the initial conditions. Hence, the system (2,4) can be integrated on taking into account (2,5), and will determine the completely defined functions V_n and V_τ, which depend on the argument β. After the integration of (2,4), on taking into account (2,5) with the known V_n and V_τ, we shall find P and ρ, adjoining to the equation (2,5) the adiabatic condition

$$\frac{p}{\rho^k} = \text{const.} \qquad (2,6)$$

This solves the above problem.

3. Application of the Generalized Flows to Developable Wings

Let EFGH be the developable surface in Fig. 1, and EH shall be the wing's frontal edge. Let us suppose that the wing meets the oncoming gas with the velocity V_1. Depending on the relative position of the wing with respect to the oncoming gas, one or two surfaces of strong disruption are formed about the edge EH (above and below), but only one sliding streamlined flow at the angle of the rotation.

Generally, the surfaces of strong disruption will not be planes. However, with properly chosen (and thus the only ones of interest) profiles of the wings the curvatures of these surfaces can be neglected; the resulting precision corresponding to the third approximation of the theory of the thin wing. We can also assume that beyond the surfaces of strong disruption the entropy is constant and whirls are absent.

The argumentation of these statements does not differ

in principle from that used in the gas dynamics of planar flows; the computation process is only slightly more complicated. With whirls absent and the entropy constant (beyond the surfaces of strong disruption) we shall have the generalized flow of Prandtl-Mayer with the cited precision.

The above is quite sufficient for evolving a scheme for the determination of the aerodynamic characteristics of the developable wing.

First, we must consider the sliding streamlining problem of the frontal edge. From the solution we shall get $V_{n,c}$, $V_{\tau,c}$, p_c and ρ_c when the flow is parallel to the surface of the wing at the frontal edge. Then, given the initial conditions $V_n=$ * etc., we must integrate the system (2,4) and solve for $V_n = V_n(\beta)$, $V_\tau = V_\tau(\beta)$, $p = p(\beta)$ и $\rho = \rho(\beta)$.
And, while working on the solution of the problem of the sliding flow over the frontal edge of the wing, one must keep in mind that V_t remains constant both during the transition over the disruption and, in that Prandtl-Mayer flow which may result at the frontal edge.

4. Certain Remarks

Note 1. In a large number of cases, the integration of the system (2,4) can be performed in series. One has only to express $d\gamma/d\beta$, V_n and V_t as series in β and to obtain from system (2,4) the coefficients of the series for V_n and V_τ.

If $d\gamma/d\beta$ is expressed in the form of a series, same results can be obtained by the method of successive approximations.

Should we use a series expansion for the flow about the wing in the presence of surfaces of strong disruption, we must delete from that series terms with higher powers of β than the third; the errors will be of order β^4.

Note 2. Should the streamlined developing surface be the surface of a circular cone, the quantity $d\gamma/d\beta$ will be constant and the integration of the system (2,4) is simplified.

Indeed, let

$$\frac{d\gamma}{d\beta} = C. \qquad (4,1)$$

Then, dividing the first equation of (2,4) by the second, we have the single differential equation:

$$\frac{dV_n}{dV_\tau} = \frac{1}{C}\frac{a}{\sqrt{V_n^2 - a^2}} - \frac{V_\tau}{V_n} \qquad (4,2)$$

Integrating (4,2) by quadratures we shall get $V_\tau(\beta)$ from the second equation of (2,4).

* $V_n = V_{n,0}$, $V_\tau = V_{\tau,0}$, $p = p_0$ и $\rho = \rho_0$, and

Note 3. The formulas become especially simple when the streamlined surface is a circular cone, and if the profile is so thin that first powers of β are sufficient. In this case, all computations become easy.

Note 4. In the case of the circular cone the magnitude $\partial\gamma/\partial\beta$ has a very simple geometrical meaning.

If M is a point on the surface of the cone, L its distance from the apex, and R the radius of curvature at M for the curve formed by the intersection of the cone with a plane passing through M and normal to the generatrix, then

$$\frac{d\gamma}{d\beta} = \frac{R}{L}. \qquad (4,3)$$

5. Flow Equations for Conic Wings

Of all developable wings the conic ones are of the greatest interest. It is therefore greatly desirable, at least for such wings, to obtain a transformation of the equations in (2,4) such as would explicitly introduce into these equations the characteristics of the conic wing's profile. This transformation will be treated in this fifth section of the author's work.

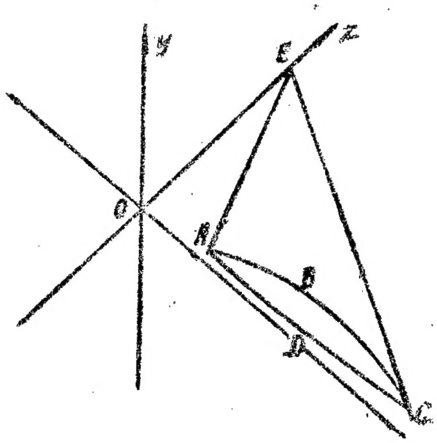

Fig. 4.

Let us suppose that ABCD is the profile of a conic wing of the points of contact with the fuselage (Fig. 4). We shall choose for the coordinate plane xy the vertical plane containing the profile ABCD. We shall direct the x axis in the direction of the unperturbed flow, the y axis vertically upward, and the z axis perpendicular to x and y.

Let us suppose that in the chosen coordinate system the apex of the cone E has the coordinates:

$$x = 0, \quad y = 0, \quad z = l. \qquad (5,1)$$

whether this apex does or does not actually belong to the wing.

And let us suppose further that the profile ABCD is given by equations

$$x = t; \quad y = \varphi(t); \quad z = 0. \qquad (5,2)$$

in the chosen coordinate system.

Then, for every value of the parameter t shall correspond a generatrix L of the cone, given by the equations:

$$\frac{x}{t}=\frac{y}{\varphi(t)}=\frac{z-l}{-l}. \tag{5,3}$$

The directional cosines of this generatrix L shall be obviously given by formulas:

$$\begin{aligned}\cos(\widehat{L,x}) &= -\frac{t}{\sqrt{t^2+\varphi^2+l^2}},\\ \cos(\widehat{L,y}) &= -\frac{\varphi(t)}{\sqrt{t^2+\varphi^2+l^2}},\\ \cos(\widehat{L,z}) &= \frac{l}{\sqrt{t^2+\varphi^2+l^2}}.\end{aligned} \tag{5,4}$$

In addition to the straight line L we shall examine the tangent T to the airfoil ABCD. This line is given by the equations:

$$\begin{aligned}y-\varphi(t) &= \varphi'(t)(x-t),\\ z &= 0\end{aligned} \tag{5,5}$$

and it has the following directional cosines:

$$\begin{aligned}\cos(\widehat{T,x}) &= \frac{1}{\sqrt{1+\varphi'^2}},\\ \cos(\widehat{T,y}) &= \frac{\varphi'}{\sqrt{1+\varphi'^2}},\\ \cos(\widehat{T,z}) &= 0.\end{aligned} \tag{5,6}$$

From the formula:

$$\cos(\widehat{T,L})=\cos(\widehat{T,x})\cos(\widehat{L,x})+\cos(\widehat{T,y})\cos(\widehat{L,y})+\cos(\widehat{T,z})\cos(\widehat{L,z}) \tag{5,7}$$

and from formulas (5,4) and (5,6) we obtain directly:

$$\cos(\widehat{T,L})=-\frac{t+\varphi\varphi'}{\sqrt{1+\varphi'^2}\sqrt{t^2+\varphi^2+l^2}}. \tag{5,8}$$

and consequently:

$$\sin(\widehat{T,L})=\frac{\sqrt{(t\varphi'-\varphi)^2+l^2(1+\varphi'^2)}}{\sqrt{1+\varphi'^2}\sqrt{t^2+\varphi^2+l^2}}. \tag{5,9}$$

We shall examine on the profile ABCD two points corresponding to the parametric values t and t + dt.

The angle between the generatrices passing through these points shall be $d\gamma$; the angle between the normals to the surface of the wings for these points shall be $d\beta$.

We shall therefore determine the two magnitudes and $d\gamma$ and $d\beta$.

If we denote the distance of a given point on the profile from the apex E of the cone by R and the distance between near points on the same profile by dS, we shall have:

$$d\gamma = \frac{\sin(\widehat{T, L})}{R} dS. \tag{5,10}$$

Since the formula for the sine $(\widehat{T,L})$ is known, and since R and dS are given by formulas:

$$R = \sqrt{t^2 + \varphi^2 + l^2}, \qquad dS = \sqrt{1 + \varphi'^2}\, dt, \tag{5,11}$$

we obtain

$$d\gamma = \Phi_1(t)\, dt, \tag{5,12}$$

where we let

$$\Phi_1(t) = \frac{\sqrt{(t\varphi' - \varphi)^2 + l^2(1 + \varphi')^2}}{t^2 + \varphi^2 + l^2}. \tag{5,13}$$

We shall now determine the second magnitude of interest, namely $d\beta$.

We introduce the unit vectors

$$\bar{L}_1 = \bar{i}\cos(\widehat{L, x}) + \bar{j}\cos(\widehat{L, y}) + \bar{k}\cos(\widehat{L, z}),$$
$$\bar{T}_1 = \bar{i}\cos(\widehat{T, x}) + \bar{j}\cos(\widehat{T, y}) + \bar{k}\cos(\widehat{T, z}), \tag{5,14}$$

where $\bar{i}$, $\bar{j}$ and $\bar{k}$ are the orthonormal of the coordinate axes.

It is obvious that the vector

$$\bar{N}_1 = \bar{T}_1 \times \bar{L}_1 \tag{5,15}$$

has the direction of the inner normal to the surface of the wing.

By using the known formulas for vector products and the previously found expressions for the direction cosines, we shall have

$$N = \sin(\widehat{T, L}) = \frac{\sqrt{(t\varphi' - \varphi)^2 + l^2(1 + \varphi'^2)}}{(1 + \varphi'^2)(t^2 + \varphi^2 + l^2)}, \tag{5,16}$$

$$N_x = \frac{l}{\sqrt{t^2 + \varphi^2 + l^2}} \; \frac{\varphi'}{\sqrt{1 + \varphi'^2}}, \tag{5,17}$$

$$N_y = -\frac{l}{\sqrt{t^2+\varphi^2+l^2}} \cdot \frac{1}{\sqrt{1+\varphi'^2}}, \qquad (5,18)$$

$$N_z = \frac{t\varphi' - \varphi}{\sqrt{t^2+\varphi^2+l^2}} \cdot \frac{1}{\sqrt{1+\varphi'^2}}. \qquad (5,19)$$

Therefore

$$\cos(\widehat{N,x}) = \frac{l\varphi'}{\sqrt{(t\varphi'-\varphi)^2 + l^2(1+\varphi'^2)}} = f_1(t),$$

$$\cos(\widehat{N,y}) = \frac{-l}{\sqrt{(t\varphi'-\varphi)^2 + l^2(1+\varphi'^2)}} = f_2(t), \qquad (5,20)$$

$$\cos(\widehat{N,z}) = \frac{t\varphi'-\varphi}{\sqrt{(t\varphi'-\varphi)^2 + l^2(1+\varphi'^2)}} = f_3(t),$$

where the functions f_1, f_2, and f_3 are merely abbreviated notations for the magnitudes in the left members.

If the formulas of (5,20) determine the direction cosines of the normal corresponding to the parameter L, it is obvious that the magnitudes

$$A_x = f_1(t) + f_1'(t)dt,$$
$$A_y = f_2(t) + f_2'(t)dt, \qquad (5,21)$$
$$A_z = f_3(t) + f_3'(t)dt$$

will be the directional cosines of the normal corresponding to the value of the parameter t + dt.

On considering the unit vectors

$$\bar{n} = \bar{i}f_1 + \bar{j}f_2 + \bar{k}f_3,$$
$$\bar{A} = \bar{i}A_x + \bar{j}A_y + \bar{k}A_z, \qquad (5,22)$$

we find easily that

$$d\beta = |\bar{n}||\bar{A}|\sin d\beta = |\bar{n} \times \bar{A}|.$$

with a precision to the small values of the higher orders.

On composing the modulus of the vector product of n and A, we obtain

$$d\beta = \Phi_2(t)\,dt, \qquad (5,23)$$

in which we let

$$\Phi_2(t) = \sqrt{(f_2f_3' - f_3f_2')^2 + (f_3f_1' - f_1f_3')^2 + (f_1f_2' - f_2f_1')^2}. \qquad (5,24)$$

Formulas (5,12) and (5,23) enable us to pass in the equations (2,4) from the independent variable β to the independent variable t.

On replacing $d\beta$ and $d\gamma$ in these equations by their expressions according to formulas (5,12) and (5,23), we obtain

the equations

$$\frac{dV_n}{dt} = \frac{aV_n}{\sqrt{V_n^2 - a^2}} \Phi_2(t) - V_\tau \Phi_1(t),$$
$$\frac{dV_\tau}{dt} = V_n \Phi_1(t), \qquad (5,25)$$

which solve the streamlining problem of the wing, whose basic principle is given by the equations (5,2).

On integrating the equations (5,25), we find V_n and V_τ as functions of the argument t.

After that, the pressure p also will be a known function of argument t.

With p known, and with expressions available for the directional cosines of the normal as well as for the magnitude $d\gamma$, the final computation of the aerodynamic factors presents no difficulties.

An analogous transformation can be made for any arbitrary, developable surface.

END